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BOUNDED COMPLEXITY JUSTIFIES COOPERATION IN THE FINITELY REPEATED PRISONERS' DILEMMA

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Cooperation in the finitely repeated prisoner's dilemma is justified, without departure from strict utility maximization or complete information, but under the assumption that there are bounds (possibly very large) to the complexity of the strategies that the players may use.

It has often been observed that cooperative behavior may emerge in non-cooperative situations when the nature of the interactions is long term. A fundamental message of the theory of repeated games is that the cooperative outcomes of multiperson games, provided those games are repeated over and over, are consistent with the usual 'selfish' utility-maximizing behavior assumed in economic theory. This message is delivered through the Folk theorem and several other results [Aumann (1959,1960,1981). Aumann and Shapley (1976), Rubinstein (1979,1980), Smale (1980) and Sorin (1983)], which assert that cooperative outcomes of the one-shot game are Nash equilibria (and also perfect equilibria) of the infinite repetition of that game. In some cases, the Nash equilibria of finitely repeated games also rationalize cooperation; this happens when the convex hull of the (vector) equilibrium payoffs contains a point that strictly dominates the smallest individually rational payoffs [Benoit and Krishna (1984)]. However, the prisoners' dilemma is not in this class of games; indeed, in any finite repetition of this game, all equilibria lead to the non-cooperative outcome at each stage. This contrasts with the common observation in the experiments involving finite repetitions of the prisoners' dilemma, that players do not always choose the single-period dominant actions, but instead, achieve some mode of cooperation.

In the literature, there are two approaches that rationalize some measure of cooperation in the finitely repeated prisoners' dilemma. Radner (1978) explores a departure from the strict 'rationality' of the Nash equilibria. If each player is satisfied to get 'close' (in utility) to the best response to the other players' strategies (ϵ-equilibria), then as the number of repetitions increases, the corresponding sets of ϵ-equilibria allow longer and longer periods of cooperation. Kreps, Milgrom, Roberts and Wilson (1982) show how incomplete information about players' options or motivation or behavior can explain the observed cooperation.

The present paper justifies cooperation in the finitely repeated prisoners' dilemma, without departure from strict utility maximization or complete information, but under the assumption that there are bounds (possibly very large) to the complexity of the strategies that the players may use.

More explicitly, if the players are restricted to using finite automata of a fixed size l, then for a sufficiently large number N of repetitions, there is an equilibrium that yields a payoff close to the

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cooperative one. This remains true even when the automaton size l is very large compared to N; indeed, l can be chosen to be an arbitrary positive power of N, which can be as large as we like.

Assume that in a (finitely or infinitely) repeated game, the actions available to player i in each stage are the elements of a finite set A^i, and the action tuples of the other players are elements of the set A^{-i}. A *finite automaton* for player i is a four-tuple $\langle M^i, q^i, f^i, g^i\rangle$, where M^i is a finite set, $q^i \in M^i$, $f^i: M^i \to A^i$, and $g^i: M^i \times \bar{A}^i \to M^i$. Intuitively, M^i is the set of possible states of the automaton, q^i is the initial state, $f^i(q)$ is the action taken by player i when in state q, and g^i describes the transition from state to state; if at state q the other players choose the action tuple a^{-i}, then the automaton's next state is $g^i(q, a^{-i})$. The *size* of the finite automaton is the number of states. Any finite automaton for player i induces a (pure) strategy in the repeated game as follows: The action to be taken by player i at stage t of the repeated game is $f^i(q^i_t)$, where $q^i_1 = q^i$, and for $t \geq 1$, $q^i_{t+1} = g^i(q^i_t, a_t^{-1})$, where a_t^{-i} are the actions of all the other players.

Consider, for instance, the repetition of the prisoners' dilemma:

	D_2	F_2
D_1	1,1	4,0
F_1	0,4	3,3

At each stage, each player i has two actions available, labelled F (Friendly) and D (Double-Cross). The strategy Tit for Tat, which begins with the friendly action and then does whatever the other player has done at the previous stage, is induced by the $\langle\{1, 2\}, 1, f, g\rangle$, where $f(1) = F$, $f(2) = D$, $g(1, F) = g(2, F) = 1$ and $g(1, D) = g(2, D) = 2$. Note also that if a strategy is induced by an automaton of size l, it can be induced by an automaton of size $l' \geqslant l$.

Let G denote the payoff function in the prisoners' dilemma. For any $N = 1, 2, \ldots$ and positive integers l_1, l_2, defined the two person game $G^N(l_1, l_2)$ as follows: The pure strategies of player i ($i = 1, 2$) are all the finite automata of size l_i. The payoff associated with a pair of pure strategies, i.e., with a pair of automata $\langle M^1, q^1, f^1, g^1\rangle$ and $\langle M^2, q^2, f^2, g^2\rangle$, is

$$\frac{1}{N} \sum_{i=1}^{N} G\left(a_t^1, a_t^2\right),$$

where $a^i_t = f^i(q^i_t)$, and $q^i_1 = q^i$, $q^i_{t+1} = g^i(q^i_t, a^{3-i})$. $G^N(l_1, l_2)$ is a 2-person game in normal form in which there are finitely many pure strategies for each player. Our first result asserts that if the sizes l_1, l_2 of the automata of the two players are between 2 and $N-1$, then there are equilibrium points that result in the friendly outcome at every stage of the game. For instance, Tit for Tat for each player is an equilibrium. If either l_1 or l_2 is at least N, then there is no equilibrium in $G^N(l_1, l_2)$ that results in the friendly outcome at every stage of the game. However, this does not exclude the possibility of cooperation, even when the automata are allowed to be very complex. Indeed, the automaton sizes may be arbitrarily large powers of the length of the game; outcomes that are arbitrarily close to the friendly outcome can still be achieved in equilibrium for sufficiently long games. For instance, if $N = 1{,}000$ and $l_1 = l_2 = 1{,}000{,}000$, then there is an equilibrium in $G^N(l_1, l_2)$ with payoffs $\geqq 2.9$ for each player. Formally, we have:

Theorem. For any integer k, there is an N_0 such that if $N \geq N_0$ and $N^{1/k} \leq \min(l_1, l_2) \leq \max(l_1, l_2) \leq N^k$, then there is a (mixed-strategy) equilibrium in which the payoffs to each player are at least $3 - 1/k$.

The proofs will appear in a subsequent paper, where we present results that are both sharper and more general, and describe in more detail the equilibrium payoffs in finite and infinite repetitions of arbitrary games, in which each player has a bound to the complexity of the strategies he may use.

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