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A COUNTEREXAMPLE TO THE FOLK THEOREM WITH DISCOUNTING

F. FORGES, J.F. MERTENS and A. NEYMAN

Catholic University of Louvain, 1348 Louvain-la-Neuve, Belgium

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The 'folk theorem' formalizes the theme that 'repetition leads to cooperation'. We present an example showing that, even with perfect monitoring, the set of Nash equilibria of the discounted games does not have to converge to the feasible, individually rational set, i.e., this version of the 'folk theorem' can break down.

A version of the folk theorem with discounting has often been mentioned in the literature [e.g., Aumann (1981)]. Sorin (1983) shows that the theorem is always true except in cases similar to our example. Counterexamples to stronger forms of the 'theorem' (involving perfectness or imperfect monitoring) have appeared in Fudenburg and Maskin (1984) and Radner et al. (1983).

The game

Player 1 chooses rows, player 2 chooses columns and player 3 chooses the matrix

$$\begin{pmatrix} 1,1,-1 & 0,0,0 \\ 0,0,\ \ 0 & 0,0,0 \end{pmatrix} \quad \begin{pmatrix} 0,0,0 & 0,\ \ 0,0 \\ 0,0,0 & 1,-1,1 \end{pmatrix}.$$

The above game is repeated, all players being informed after every stage of all pure strategy choices.

Players 2 and 3 can each guarantee themselves 0, and as the sum of their payoffs is 0, all equilibria of the discounted game and of the undiscounted one are of the form $(x, 0, 0)$. The 'folk theorem' asserts that the equilibria of the undiscounted game are $(x, 0, 0)$ $(0 \leqslant x \leqslant 1)$.

But only $(0, 0, 0)$ is an equilibrium of the discounted games (or of the n-stage games): otherwise there would be a first stage, say n_0, where either $(1, 1, -1)$ or $(1, -1, 1)$ is hit with positive probability under the equilibrium – say $(1, 1, -1)$ is. Then player 2 gets a positive payoff by playing the equilibrium before n_0, and left from n_0 on. Since in any equilibrium player 2 gets only zero, we have a contradiction.

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