# On the Popular Support for Progressive Taxation

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## Abstract

The "popular support for progressive taxation theorem" (Marhuenda and Ortuño-Ortín, 1995) provides an important formalization of the intuition that a majority of relatively poor voters over rich ones leads to progressive income taxation. Yet the theorem does not provide an equilibrium outcome. In addition, it assumes an overly restrictive domain of tax schedules and no incentive effects of income taxation. This paper shows that none of these assumptions of the theorem can be relaxed completely. Most notably, it is shown that a majority of poor voters does not imply progressive taxation in a more general policy space and that a regressive tax schedule may obtain a majority over a progressive one when individuals' income is endogenous.

## 1. Introduction

It is a well established regularity that democracies show a strong revealed preference for increasing marginal statutory income tax rates. Theoretical studies of this fact are, however, still largely inconclusive; the related analyses are confined to either linear or quadratic tax schedules for technical reasons. Hence, they are of considerably limited descriptive content. With a general domain of tax schedules the main message appears to be that "self-interested voting would lead to gross instability and cycling over tax structures, with new majority coalitions perpetually emerging and overturning the existing tax code in favor of a new one which favors them. This picture of perpetual chaos is, again, hardly plausible empirically" (Kramer 1983, p. 226).

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Recently, however, in an interesting paper Marhuenda and Ortuño-Ortín (1995) showed that if the median voter's income level is below the mean income of the population, any marginal-rate progressive tax would always have the majority support over any marginal-rate regressive tax (which collects the same revenue), provided that the former treats the poorest individual not worse than the latter. This is a promising development which captures the intuition that, whenever the number of relatively poor voters exceeds the number of richer voters, only progressive policies are consistent with majority voting.

Unfortunately there are severe difficulties in embedding this result in an equilibrium framework. Perhaps more importantly, the analysis of Marhuenda and Ortuño-Ortín (1995) is restricted only to convex and concave functions.[1] According to the intuition above, one must be able to generalize the result for the class of all average-rate progressive and all average-rate regressive taxes.[2] In other words, it seems intuitive that the presence of a majority of relatively poor voters would imply that every average-rate progressive tax schedule beats every average-rate regressive tax schedule under pairwise majority voting. In this paper I show that, surprisingly, this is not the case. In fact, even a marginal-rate regressive tax schedule may prevail over an average-rate progressive tax in pairwise majority voting.

A second shortcoming of the "popular support for progressive taxation" theorem is its abstention from incentive effects. It is shown here that introducing a preference for leisure in the individuals' utility function has a great impact on their decisions with respect to the desired tax schedule. When income is endogenously determined, even low productivity individuals may favor less progressive taxes if high marginal income taxes will cause large deadweight losses. In particular I show here that, in an economy with endogenous income, a linear tax may obtain a majority over a marginal-rate progressive one. Hence, the claim that "the more poor people are in the society, the larger will be the support for progressive taxation" is found misleading also in this case.

These negative results depict the limitations of the standard direct democracy approach as a means to formalizing the regularity that all developed countries implement marginal-rate progressive income tax schedules. It seems, therefore, that embedding the intuition behind the direct democracy model in a model of representative democracy may be more promising to explain the empirically observed demand for progressive taxation.

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[1]Note that a tax schedule is marginal-rate progressive (regressive) if, and only if, it is a convex (concave) function.

[2]Formally, a tax schedule is said to be *average-rate progressive (regressive)* if the mapping $y \rightarrow t(y)/y$ is increasing (decreasing). Note that every marginal-rate progressive (regressive) tax schedule is average-rate progressive (regressive) but not conversely.

## 2. The Popular Support for Progressive Taxation Theorem

The framework I employ is the same as in Marhuenda and Ortuño-Ortín (1995). The pre-tax income distribution of the economy is described by a continuous and strictly increasing distribution function $F$ on $[0, 1]$, where each individual is identified by her income $y$ in $[0, 1]$. Define $\mu$ and $m$ as the mean and median income of $F$, respectively; that is, $\mu \equiv \int_0^1 y \, dF$ and $m \equiv F^{-1}(1/2)$. I assume throughout that $m < \mu$.

A tax schedule is a continuous and increasing function $t : \mathbf{R}_+ \to \mathbf{R}$ such that $t(y) \leq y$ for all $y \in [0, 1]$ and

$$\int_0^1 t(y) \, dF = R,$$

where $R \in (0, \mu)$ is a predetermined level of aggregate fiscal revenue that has to be collected. Let $\mathcal{T}$ denote the set of tax schedules that satisfy the above conditions and are either convex or concave functions.

Consider two tax schedules in $\mathcal{T}$, $t_1$ convex and $t_2$ concave, $t_1 \neq t_2$, with $t_1(0) \leq t_2(0)$. Under the previous assumptions it is not difficult to show that there exists an income $y^* \in (0, 1)$ such that

1. $t_1(y) < t_2(y)$, for $y \in [0, y^*)$;
2. $t_1(y^*) = t_2(y^*)$;
3. $t_1(y) > t_2(y)$, for $y \in (y^*, 1]$.

The following result (due to Marhuenda and Ortuño-Ortín, 1995) states that the income of the individual who is indifferent between the two taxes is greater than the median income.

PROPOSITION 1: $m < y^*$.

This implies that if individuals vote for the tax schedule that minimizes their tax liability, under majority voting any marginal-rate progressive tax policy prevails over any marginal-rate regressive one, provided that the first treats the poorest individual in the society no worse than the second.

Proposition 1 is an important result that establishes that the set of marginal-rate progressive taxes is stable in $\mathcal{T}$ under majority voting. Thereby it rules out the implementation of a marginal-rate regressive tax schedule in equilibrium. Yet several critical questions remain open: what is the equilibrium outcome of the election? What happens if we allow political parties to propose taxes outside $\mathcal{T}$? Does the proposition remain valid when the individuals' income is endogenous? These issues are explored next.

## 3. Generalizations of the Model

### 3.1. Equilibrium Analysis

Consider the following strategic game. There are two political parties; parties propose simultaneously and independently an income tax policy from $\mathcal{T}$. It is assumed that individuals vote sincerely and indifferent voters abstain from voting. Both political parties know the income distribution $F$ and their only objective is to win the election by selecting a fiscal policy preferred by a majority to the one selected by the other party.

Given the two proposals made by the parties, $t_i$, $i \in \{1, 2\}$, define the function $\varphi_{t_1,t_2} : [0, 1] \to \{0, 1, 2\}$, with $\varphi_{t_1,t_2}(y) = 0$ if $t_1(y) = t_2(y)$, $\varphi_{t_1,t_2}(y) = 1$ if $t_1(y) < t_2(y)$, and $\varphi_{t_1,t_2}(y) = 2$ if $t_1(y) > t_2(y)$.

Given $\varphi_{t_1,t_2}$, the share of votes obtained by party 1 is

$$N(t_1, t_2) = F\big(\varphi^{-1}_{t_1,t_2}(1)\big).$$

Party 1 wins the election if $N(t_1, t_2) > N(t_2, t_1)$ and loses it if $N(t_1, t_2) < N(t_2, t_1)$. Whenever $N(t_1, t_2) = N(t_2, t_1)$, each party wins with equal probability.[3] Considering that each party is only interested in winning and can choose among the same set of admissible policies, we have a symmetric two-player zero-sum game $G$, where the action space of each party is $\mathcal{T}$, and the utility function of party 1 is

$$U(t_1, t_2) = \begin{cases} 1 & \text{if} \quad N(t_1, t_2) > N(t_2, t_1), \\ 0 & \text{if} \quad N(t_1, t_2) = N(t_2, t_1), \\ -1 & \text{if} \quad N(t_1, t_2) < N(t_2, t_1). \end{cases}$$

$G$ has a Nash equilibrium in pure strategies if and only if there exists a Condorcet winner; that is, a policy $t^* \in \mathcal{T}$ such that $N(t^*, t) > N(t, t^*)$ for all $t \in \mathcal{T}\backslash\{t^*\}$. It is well known that such a $t^*$ does not exist; given a tax schedule, it is always possible to design an alternative that hurts a minority of individuals but benefits the rest, thus defeating the first tax in pairwise majority voting.

PROPOSITION 2: *For every $t \in \mathcal{T}$, there exists $t' \in \mathcal{T}$ with $N(t', t) > N(t, t')$.*

This result is well known in the folklore (see, for example, Marhuenda and Ortuño-Ortín, 1998), yet, to the best of my knowledge, a formal proof has not appeared in print. For completeness, I thus offer a proof in the Appendix of this paper.

In passing, I should mention that while nonexistence of a pure strategy Nash equilibrium is an obvious shortcoming of this approach, Carbonell-Nicolau and Ok (2000) have recently shown that mixed strategy equilibria of

[3]Since indifferent individuals abstain from voting, the measure of $\varphi_{t_1,t_2}(y) = 0$ is irrelevant when determining the winner of the election. Given that $\varphi_{t_1,t_2}(y) = 0$ is a measure zero event in all the cases presented below, this assumption is without loss of generality.

$G$ exist in a variety of cases. They also show that, when such a mixed strategy equilibrium of $G$ exists, the support of the equilibrium strategies of the parties consists only of marginal-rate progressive taxes. Yet, as I shall show below, this result is not robust relative to very mild and realistic expansions of the policy space $\mathcal{T}$.

### 3.2. Average-Rate Progressive Taxes

An obvious shortcoming of Proposition 1 is the arbitrary restriction of the policy space to marginal-rate progressive and regressive tax schedules. It is difficult to justify this restriction, especially since nonconvex–nonconcave tax schedules are sometimes (albeit, rarely) seen to be implemented in practice.[4] So it is natural to ask whether "the support for progressive taxation" theorem remains valid when we include in the policy space the class of all average-rate progressive tax schedules. Unfortunately, the following example provides a negative answer.[5]

*Example 1:* Let the distribution function $F$ be given by

$$F(y) = 2y - y^2 \quad \text{for} \quad y \in [0, 1].$$

The mean income $\mu$ is then $1/3$, which is greater than the median income $m \approx 0.292$. Consider the following three-bracket average-rate progressive tax schedule

$$t_p(y) = \begin{cases} \alpha y & \text{for} \quad 0 \leq y < 1/8, \\ (1-\alpha)(y - 1/8) + (1/8)\alpha & \text{for} \quad 1/8 \leq y < 1/4, \\ (1/2)y & \text{for} \quad 1/4 \leq y \leq 1, \end{cases}$$

where $\alpha \in (0, 1/2)$. It is easily checked that $t_p$ is average-rate progressive on $[0, 1]$; that is, $t_p(y)/y$ is increasing on $[0, 1]$ (see Figure 3.1). For $y \in [0, 1/4]$ we have

$$t_p(y) < \frac{1}{2}y, \quad 0 < y < \frac{1}{4}. \tag{3.1}$$

Take now the following regressive tax schedule

$$t_r(y) = \begin{cases} (1/2)y & \text{for} \quad 0 \leq y < 1/4, \\ \beta(y - 1/4) + 1/8 & \text{for} \quad 1/4 \leq y \leq 1, \end{cases}$$

[4] For example, from 1988 to 1991, the statutory federal income tax schedule in the U.S. was average-rate progressive, but not marginal-rate progressive due to the non-monotonicity of its top three brackets (see Mitra and Ok 1996).

[5] This example is based on a modification of an example kindly communicated to me by Tapan Mitra.

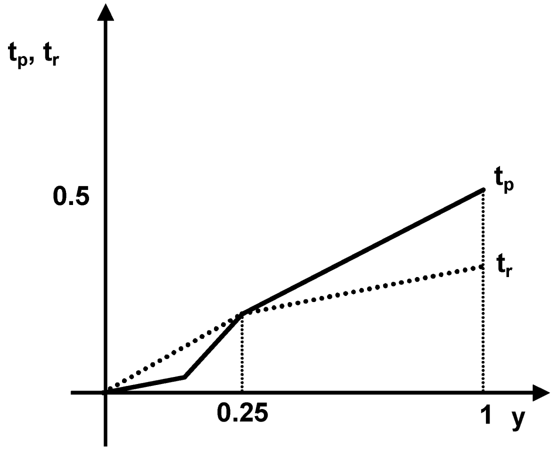


Figure 3.1: An average-rate progressive versus a marginal-rate regressive tax schedule

with $\beta \in (0, 1/2)$. Letting $\varepsilon := 1/2 - \beta > 0$, we may write $t_r(y) = (1/2 - \varepsilon)(y - 1/4) + 1/8 = 1/2y - \varepsilon(y - 1/4)$ for $y \in [1/4, 1]$. Thus,

$$t_r(y) < \tfrac{1}{2}y, \quad 1/4 < y \leq 1.$$

Combining this with (3.1), we find that

$$t_p(y) < t_r(y) \quad \text{for} \quad y \in [0, 1/4),$$

and

$$t_r(y) < t_p(y) \quad \text{for} \quad y \in (1/4, 1].$$

Given that $m > 1/4$, this implies that the regressive tax $t_r$ will win in a majority vote over the progressive tax $t_p$.[6]

This example shows that Proposition 1 fails when $\mathcal{T}$ is large enough to include average-rate progressive taxes. In passing, we note another surprising implication of this example. As shown by Fellman (1976) and Jakobsson (1976), income inequality decreases (in the sense of relative Lorenz dominance) when an average-rate progressive tax is implemented, while income inequality increases under an average-rate regressive tax. So, had Proposition 1 remained valid for the enlarged set of feasible tax schedules, we could have concluded that democracies show a revealed preference for equality. The previous example, however, shows that this is not the case. Since the regressive tax schedule $t_r$ prevails over the progressive one $t_p$, it is as if the society reveals a social preference for more inequality in this case, even though the income of the majority of the population is below the mean, a paradoxical observation.

[6]To complete the example we need to fix the values of $\alpha$ and $\beta$ such that the two taxes collect the same revenue. If we set $R = 0.16$, the tax schedules satisfy the revenue constraint when $\alpha = \frac{807}{3150}$ and $\beta = \frac{611}{1350}$.

### 3.3. Endogenous Income

This section analyzes the case in which individuals react (through their consumption-leisure choices) to the implemented tax schedule. Incentives complicate the picture since now individuals vote for a tax schedule and, after a tax schedule has been implemented, they choose their optimal pre-tax income level.[7]

In this scenario individuals are no longer represented by their incomes but by their ability level $a \in [0, 1]$. I assume that all the individuals have the same utility function $U(C, L)$, which is a strictly quasi-concave, twice continuously differentiable function of consumption, $C > 0$, and labor, $L \geq 0$, with $\partial U/\partial C > 0$ and $\partial U/\partial L < 0$. Individual $a$'s pre-tax income is $Y = aL$. Given a tax schedule $t$ in $\mathcal{T}$ her consumption is $C = Y - t(Y)$. Here I let $F$ stand for the distribution of abilities instead of incomes; as before, $F$ is assumed to be continuous and strictly increasing, with its mean strictly greater than its median.

In this case, given that indifference curves are well behaved and preferences are smoothly distributed, we observe bunching of individuals with different ability levels at convex kink points of the budget set; consequently, one can construct examples in which Proposition 1 no longer holds.

To see this, consider the unique optimal labor supply of each individual with ability level $a$, $L^*(a, \Theta)$, when the tax schedule is

$$t(Y, \Theta) = \begin{cases} t_1 Y, & \text{if} \quad Y \leq \overline{y}, \\ t_1 \overline{y} + t_2(Y - \overline{y}), & \text{if} \quad Y > \overline{y}, \end{cases}$$

where $\Theta = (t_1, t_2, \overline{y})$ denotes the tax parameters, $t_1 < t_2$, and $\overline{y} > 0$. For this tax schedule the individuals' budget sets are convex, which guarantees that for each $a$ there exists a unique optimum. By examining the relevant Kuhn-Tucker conditions, one can show that there exist two functions $\phi : [0, 1]^2 \rightarrow \mathbf{R}_+$ and $\varphi : [0, 1]^3 \rightarrow \mathbf{R}_+$ such that

$$L^*(a, \Theta) = \begin{cases} \phi(a, t_1) & \text{if} \quad a \leq a_1, \\ \overline{y}/a & \text{if} \quad a_1 < a < a_2, \\ \varphi(a, t_1, t_2) & \text{if} \quad a_2 \leq a, \end{cases}$$

with

$$\phi(a_1, t_1) = \frac{\overline{y}}{a_1} \quad \text{and} \quad \varphi(a_2, t_1, t_2) = \frac{\overline{y}}{a_2}.$$

[7]Several papers investigate the connection between endogenous income and income taxation by placing strong restrictions on the set of feasible tax schedules. Romer (1975), Roberts (1977), and Meltzer and Richard (1981) allow only linear tax schedules, while Cukierman and Meltzer (1991) and De Donder and Hindricks (2000) allow only for quadratic functions. The framework I adopt is the standard one used in all of these papers except that I allow here for a more general set of feasible tax schedules.

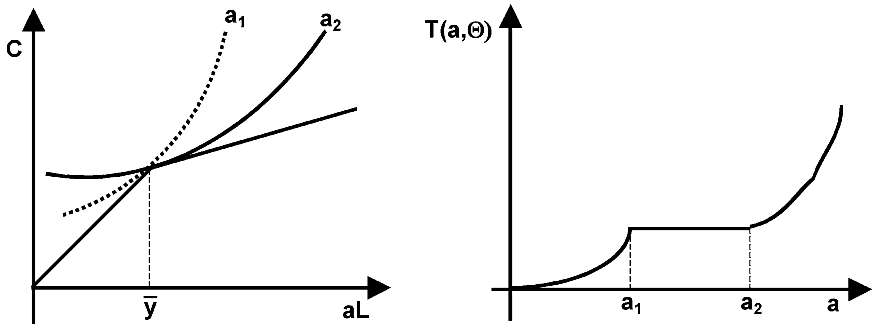


Figure 3.2: A two-bracket tax schedule and its tax liability function

Once individuals solve their optimization problems, we obtain a tax liability function $T(a, \Theta) \equiv t(aL^*, \Theta)$ that depends only on the individuals' ability level and the tax parameters. It is important to observe that, because there is bunching of abilities at the kink point of the budget sets, $T(a, \Theta)$ presents a flat region for any $a$ between $a_1$ and $a_2$; thus, the tax schedule is convex but the tax liability function is not (Figure 3.2).[8]

As a consequence of this, there exist concave tax schedules that defeat convex tax schedules even for right-skewed distributions of abilities. To make things precise, I offer the following example.[9]

*Example 2:* Individuals' utility function is given by

$$U(C, L) = \frac{1}{2}\ln C + \frac{1}{2}\ln(1 - L),$$

and the distribution of abilities is given by

$$F(a) = 2a - a^2 \quad \text{for } a \in [0, 1].$$

Due to the revenue constraint there exists a unique linear tax schedule, the tax rate $t_l$ of which satisfies

$$t_l \int_0^1 aL^*(a, t_l)\, dF(a) = t_l \times \frac{1}{6} = R.$$

Fixing $R = 0.05$ implies that $t_l = 0.3$. Under this tax schedule the indirect utility function of an individual with ability $a$ is

$$V(a, t_l) = \frac{1}{2}\ln(0.35a) + \frac{1}{2}\ln\frac{1}{2}.$$

[8]Note that the particular shape of the increasing parts of the tax liability function is determined by the individuals' utility function.

[9]I thank an anonymous referee for providing several suggestions that greatly improved the exposition of this example.

Consider now the following convex tax

$$t_c(Y, \Theta) = \begin{cases} 0, & \text{if} \quad Y \leq 0.05 \\ 0.48281(Y - 0.05), & \text{if} \quad Y > 0.05, \end{cases}$$

that satisfies the budget constraint

$$0.48281 \int_{a_2}^{1} \left[aL^*(a, t_c) - 0.05\right] dF(a) = R = 0.05.$$

For all the individuals with $a \leq a_2 = 0.14668$ their budget set under $t_c$ contains their budget set under $t_l$. Therefore, these individuals prefer $t_c$ to $t_l$. This is not the case for all the individuals with ability level above $a_2$. These individuals' indirect utility function is given by

$$V(a, t_c) = \frac{1}{2} \ln \left[ \frac{a(1 - t_2)}{2} + \frac{\overline{y} t_2}{2} \right] + \frac{1}{2} \ln \left[ \frac{1}{2} + \frac{\overline{y} t_2}{2a(1 - t_2)} \right].$$

An individual with ability $a$ prefers $t_l$ over $t_c$ if and only if $V(a, t_l) > V(a, t_c)$. That is, if

$$\ln(0.35a) + \ln \frac{1}{2} - \ln \left[ \frac{a(1 - t_2)}{2} + \frac{\overline{y} t_2}{2} \right] - \ln \left[ \frac{1}{2} + \frac{\overline{y} t_2}{2a(1 - t_2)} \right] > 0.$$

Routine calculations show that the above inequality is satisfied for every $a > 0.2857$. Since the median ability level is 0.2928 we obtain that a majority of the population prefers a linear tax schedule to a progressive tax schedule.

Example 2 shows that Proposition 1 ceases to hold true in the presence of incentive effects. More importantly, this example points to the crux of the problem: the potential bunching of some individuals with different ability levels at kink points of the budget set. In particular, we learn from this example that to generalize the result of Marhuenda and Ortuño-Ortín (1995) to an economy with incentives, we need to impose a further condition on $\mathcal{T}$: the differentiability of the tax schedules. If tax schedules are differentiable there are no kink points in the individuals' budget sets, and consequently there is no bunching of abilities. In this framework, if the individuals' utility function is such that the income effect and the substitution effect cancel each other out (as would be the case with Cobb-Douglas utility functions), then the tax liability function preserves the convexity or concavity of the tax schedule, and we recover Proposition 1.[10] Generalizations of this property to larger classes of utility functions appear worthy of future research.

[10]To see this, approach any differentiable tax schedule using a piecewise linear function. As the number of brackets increases, the measure of abilities bunching at kink points decreases. When the number of brackets goes to infinity, the measure of abilities bunching at kink points goes to zero.

## 4. Conclusion

This paper studied three possible extensions of the "popular support for progressive taxation" theorem. This theorem formalizes the intuition that, whenever the number of relatively poor voters exceeds the number of rich voters, only progressive policies are consistent with majority voting. Two critical assumptions of the theorem are the restrictive set of feasible tax schedules and the exogeneity of the individuals' income. This paper showed that a majority of poor voters does not necessarily imply a majority support for progressive taxation when the analysis is extended even only to include the class of all average-rate progressive tax schedules. Furthermore, even in a restricted domain of tax schedules, a linear tax schedule obtains a majority over a progressive one when income is endogenous.

These results highlight some limitations of the standard direct democracy approach when trying to explain the empirically observed demand for progressive taxation. There exist, however, alternative strategies that may allow us to model such a strong empirical regularity in a general manner. For example, imposing other restrictions on the set of feasible tax schedules; or assuming a different voting behavior of the population (Chen 2000); or embedding Marhuenda and Ortuño-Ortín's result in a representative democracy framework (Carbonell-Nicolau and Klor 2003). These different approaches have delivered positive preliminary results.

## Appendix

*Proof of Proposition 2:* From Proposition 1 we know that $N(t', t) > N(t, t')$ for every concave $t$ and convex $t'$ in $\mathcal{T}$ with $t \neq t'$. Therefore, we only need to concentrate on convex $t$. The proof proceeds by construction, differentiating between two main cases.

Suppose first that $t$ is such that $t(1) < 1$. In this case, consider the following tax schedule:

$$t_1(y) = \max\left[t(y) - \varepsilon_1,\, t(b) - \varepsilon_1 + \frac{1 - t(b) + \varepsilon_1}{1 - b}(y - b)\right],$$

with $b > m$ and $\varepsilon_1 > 0$ such that

$$\int_0^b [t(y) - \varepsilon_1]\, dF + \int_b^1 \left[t(b) - \varepsilon_1 + \frac{1 - t(b) + \varepsilon_1}{1 - b}(y - b)\right] dF = R.^{11}$$

[11] Such an $\varepsilon$ is uniquely well defined for any given $b$. To see this define $\psi(\varepsilon) := \int_0^b [t(y) - \varepsilon]\, dF + \int_b^1 [t(b) - \varepsilon + \frac{1 - t(b) + \varepsilon}{1-b}(y - b)]\, dF$. Note that $\psi$ is continuous in $\varepsilon$. Since $\psi(0) > R$ and $\psi(\overline{\varepsilon}) < R$ for $\overline{\varepsilon}$ big enough, by the intermediate value theorem there exists $\varepsilon_1$ such that $\psi(\varepsilon_1) = R$. Since $\psi$ is strictly decreasing in $\varepsilon$, $\varepsilon_1$ is uniquely defined.

Since $t_1$ is convex and satisfies the revenue constraint, $t_1 \in \mathcal{T}$; given that $b > m$, clearly $N(t_1, t) > 1/2 > N(t, t_1)$.

Suppose now that $t$ is such that $t(1) = 1$. In order to proceed with this case, it is helpful to define the following tax schedule:

$$t_m(y) = \max\left[\alpha, \alpha + \frac{1-\alpha}{1-m}(y-m)\right],$$

where $\alpha$ is obtained from the revenue constraint

$$\int_0^m \alpha \, dF + \int_m^1 \left[\alpha + \frac{1-\alpha}{1-m}(y-m)\right] dF = R.^{12}$$

In words, $t_m$ is the tax schedule preferred by the individual with the median income; that is, $t_m(m) \leq t(m)$ for any $t \in \mathcal{T}$. Note that this tax is uniquely defined.

If $t(0) > t_m(0)$, since $t$ and $t_m$ are nondecreasing and $t_m(m) = t_m(0) = \alpha < t(0)$, it follows that $t_m(y) < t(y)$ for every $y \in [0, m]$. Therefore, $N(t_m, t) > 1/2 > N(t, t_m)$.

If $t(0) \leq t_m(0)$, $t \neq t_m$, we can construct the following tax schedule:

$$t_2(y) = \max\left[t(c) - \varepsilon_2, t(y) - \varepsilon_2\right],$$

with $c < m$ and $\varepsilon_2 > 0$ such that the revenue constraint is satisfied. Since $t_2(y) < t(y)$ for every $y \in [c, 1]$ and $c < m$, then $N(t_2, t) > 1/2$.

Finally, for $t = t_m$, consider the following tax schedule:

$$t_3(y) = \begin{cases} \alpha & \text{for} \quad y \in [0, m-d], \\ \alpha + \frac{t(m+d) - \varepsilon_3 - \alpha}{2d}(y - m + d) & \text{for} \quad y \in (m-d, m+d), \\ t_m - \varepsilon_3 & \text{for} \quad y \in [m+d, 1], \end{cases}$$

where $\varepsilon_3$ is such that the budget constraint is satisfied (for a fixed $d$). Note that all the individuals with income between 0 and $m-d$ are indifferent between the two tax schedules; individuals with income in $(m-d, x)$ vote for $t_m$ and individuals with income greater than $x$ vote for $t_3$, where $x$ solves

$$\frac{t(m+d) - \varepsilon_3 - \alpha}{2d}(x - m + d) = \frac{1-\alpha}{1-m}(x-m).$$

Since the income distribution $F$ is assumed continuous, as $d \to 0$, we get $N(t_3, t) > 1/2 > N(t, t_3)$. ■

[12]The same argument used in the previous footnote can be applied here to show that $\alpha$ is uniquely well defined.

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