Creation of discrete quasienergy resonance states in strong electromagnetic fields

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The combined complex-coordinate-Floquet formalism, which has been described in detail in J. Chem. Phys. **94**, 7311 (1991), is used for numerical studies of the creation of discrete long-lived quasienergy resonance states for two model Hamiltonians which describe the interaction of dissociative/ionizing systems with intense monocromatic field. Despite the fact that the two model Hamiltonians have the same energy spectrum in the absence of field, the number of resonance states which they support and the general behavior of the resonances in the presence of the field is entirely different. An analysis of the dependence of the quasienergy resonance spectrum on field intensity in terms of the dependence of the dressed potential spectrum in the field intensity was found adequate for medium intensity fields, but failed for strong fields.

I. INTRODUCTION

When the Hamiltonian describing a physical system is time independent, the time dependent Schrödinger equation (for $\hbar=1$)

$$i\frac{\partial\psi}{\partial t}=\hat{H}\psi \tag{1}$$

has stationary solutions. For a dissociative system, some of these solutions are bound states and some are continuum states.

When the (dissociative) system interacts with some radiation source with a period T (e.g., a cw laser)

$$\hat{H}(t)=\hat{H}(t+nT), \quad n=0,\pm1,\pm2,\pm3,..., \tag{2}$$

the Hamiltonian is no longer time independent, and stationary solutions of the time dependent Schrödinger equation cease to exist. For this case, however, the Floquet theorem[1] suggests the existence of quasistationary states (known also as Floquet states) Φ_α and (time independent) quasienergies ϵ_α such that a possible solution of the time dependent Schrödinger equation is given by

$$\psi_\alpha=e^{-i\epsilon_\alpha t}\Phi_\alpha, \tag{3}$$

where the quasienergy states are time periodic

$$\Phi_\alpha(t)=\Phi_\alpha(t+nT). \tag{4}$$

The interaction with the periodic field turns the bound states of the interactionless dissociative system into metastable resonance states having finite lifetimes. These resonance states can be characterized by complex quasienergies

$$\epsilon_\alpha=\epsilon_\alpha(\mathrm{pos})-i\frac{\Gamma_\alpha}{2}, \tag{5}$$

where $\epsilon_\alpha(\mathrm{pos})$ is the resonance position and Γ_α is the resonance width which is inversely proportional to the lifetime of the resonance state. By substituting Eq. (5) into Eq. (3), one can see that the wave function corresponding to the resonance $\psi_\alpha(\mathbf{x},t)$ vanishes as $t\to\infty$. In order to conserve probability in coordinate and time space, $\Phi_\alpha(\mathbf{x},t)$ should exponentially diverge as $x\to\infty$. The desired bound-like square-integrable boundary condition can be obtained by complex scaling the coordinates according to[2]

$$\mathbf{x}\to\mathbf{x}e^{i\theta}. \tag{6}$$

and following the Balslev–Combes theorem[3]

$$\Phi_\alpha(\mathbf{x}e^{i\theta})\to0 \tag{7}$$

as $x\to\infty$.

From perturbative considerations, it is expected that we find a one-to-one correspondence between the number of bound states of the field-free system and the number of resonance states which exist in the presence of the monochromatic electromagnetic field. Indeed this is the situation in weak fields.

For stronger field intensities, however, the number of resonance states is not identical to the number of bound states, and the phenomenon of either "creation"[4–7] or "annihilitation"[8] of states has been observed. The numerical evidence that the number of discrete long-lived quasienergy states is *NOT* necessarily equal to the number of bound states of the field-free Hamiltonian, is a striking manifestation of the nonlinear processes induced by the intense field.

In this work, we study the creation and annihilation of quasienergy resonance states in two 1D model potentials which have the same energy spectrum—a periodically driven Morse potential and a periodically driven inverted-Gaussian potential. The latter model was also studied by Bardsley, Szöke, and Comella,[5] who found for this system the creation of "extra" quasienergy resonance states having extremely short lifetimes (these are, therefore, nonobservable resonances). These authors have predicted[5] that long-lived "extra" resonances should appear as field intensity is increased. This prediction is indeed verified in our calcu-

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lations. Dörr and Potvliege[9] have shown that one of the "extra" resonances can be traced to a shadow pole of one of the bound states of the field-free Hamiltonian. More recently, Yau and Chu[10] studied the same model by the complex-scaling Fourier-grid Hamiltonian method in coordinate representation developed by Chu.[11] They found oscillatory structures of the photodetachment rate as a function of field strength, revealing that the decreasing of the ionization rate may not necessarily be a monotonic behavior. Although this subject is out of the scope of our present study, we should mention that in our representation of the resonance complex eigenvalues λ_α of the evolution operator for one optical cycle, the resonance width (i.e., ionization rate) dependence on the field strength is shown to be associated with an avoided crossing in the complex λ plane.

Although the formalism which we used to find the resonance quasienergies is not different from the one used by Bardsley, Szöke, and Comella[5] (the only differences are in the computational methods used to obtain the evolution operator and in the representation of the results), for the sake of clarity we provide a brief description of the method in Sec. II. Creation of new long-lived resonance quasienergy states for the periodically driven inverted-Gaussian potential (in addition to those obtained by Bardsley *et al.*[5]) are given in Sec. III. The creation and annihilation of resonance quasienergy states for the periodically driven Morse potential are given in Sec. IV. Section V pertains to concluding remarks emphasizing the advantages in the representation of resonance quasienergy states in the complex λ plane in the study of annihilation, creation, and avoided crossings phenomena.

II. THE COMPLEX SCALED EVOLUTION OPERATOR

The definition of the evolution operator[12] $\hat{U}_\theta(t|0)$ is not altered by complex scaling such that

$$\psi_\alpha(\mathbf{x}e^{i\theta}) = \hat{U}(t|0)\psi_\alpha(\mathbf{x}e^{i\theta},0) \tag{8}$$

substituting Eqs. (3) and (4) into Eq. (8) and setting the time to one period ($t=T$) leads directly to the eigenvalue equation

$$\hat{U}_\theta(T|0)\Phi_\alpha(\mathbf{x}e^{i\theta},0) = \lambda_\alpha\Phi_\alpha(\mathbf{x}e^{i\theta},0) \tag{9}$$

with ($\hbar=1$)

$$\lambda_\alpha = e^{-i\epsilon_\alpha T}. \tag{10}$$

An eigenvalue λ_α of the evolution operator for one optical cycle $\hat{U}_\theta(T|0)$ is either associated with a quasienergy resonance state or with a quasienergy continuum state. The Balslev–Combes theorem[3] ensures that resonance states are θ independent (for $\theta > \theta_c$)

$$\lambda_\alpha = \exp[-i\epsilon_\alpha(\mathrm{pos})T]e^{-\Gamma_\alpha T/2} \tag{11}$$

and that continuum states are θ dependent in the form

$$\lambda_\alpha = \exp(-i\epsilon_\alpha e^{-i2\theta}T). \tag{12}$$

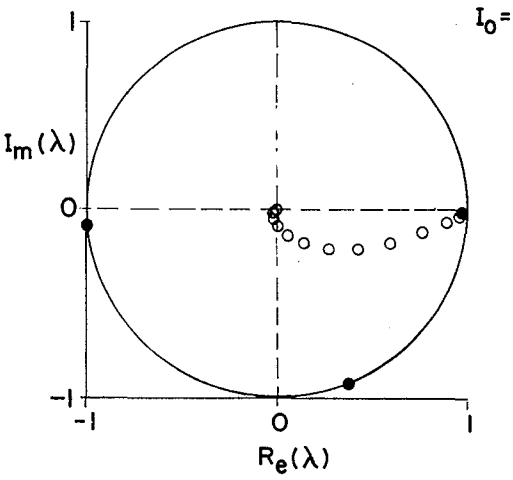

FIG. 1. The bound states and the rotating continua E_0 of the complex-scaled Gaussian potential [Eq. (14) for $I_0=\mu_0=0$] given by $\lambda=\exp(-iE_0T/\hbar)$ when T is the period of the monochromatic field that is turned on when $\mu\neq 0$. λ is given in dimensionless units.

From Eqs. (11) and (12), one can see that all eigenvalues λ_α are restricted to an area circumscribed by the unit circle in the complex λ plane (see, e.g., Fig. 1). From Eq. (12), one can clearly see that the eigenvalues λ_α, which are associated with continuum states, form a spiral (the empty circles in Fig. 1) which converges to the origin ($\lambda=0+i0$). This convergence becomes more rapid as the scaling angle θ is increased. From Eq. (11), one can see that the polar angle in the λ plane is determined by the resonance position $\epsilon_\alpha(\mathrm{pos})$ (modulus ω), and that the absolute value $|\lambda_\alpha|$ is determined by the resonance width Γ_α. Bound states having zero width (infinite lifetime), therefore, are associated with eigenvalues λ_α, which lie on the unit circle (e.g., the three full circles in Fig. 1), whereas resonance states with broad widths (short lifetimes) are associated with eigenvalues located close to the origin ($\lambda_\alpha=0+i0$).

The calculation procedure involves solving the evolution equation

$$i\dot{U}_\theta(t|0) = \hat{H}(\mathbf{x}e^{i\theta},t)U_\theta(t|0) \tag{13}$$

for the known Hamiltonian, and with the initial condition $U_\theta(0|0)=I$ (where I is the unit matrix) to obtain $U_\theta(T|0)$. This matrix is subsequently diagonalized to obtain the eigenvalues λ_α [Eq. (9)].

The propagation of the complex-scaled time evolution operator to obtain $U_\theta(T|0)$ was carried out as described in Ref. 13 by taking the two leading terms in the Magnus series expansion of the time-evolution operator, exponentiating the Hamiltonian up to the first five terms in the Taylor expansion, and using a discrete representation (128 to 512 grid points), both in configuration space and via the fast Fourier transform (FFT) also in momentum space. The coordinates were complex scaled by $\exp(i\theta)$. Stationary complex quasienergy resonance solutions for which $\partial E_{\mathrm{res}}/\partial\theta=0$ were obtained for $\theta=0.3$, 0.5, and 0.7 rad. In our calculation, the real and imaginary parts of the com-

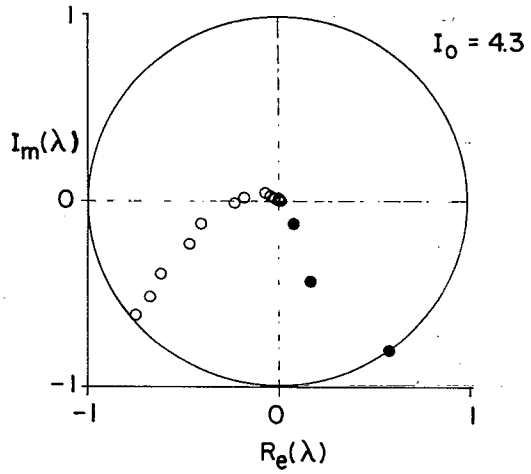

FIG. 2. The resonance quasienergy levels (full circles) and the rotating continua of the periodically driven Gaussian potential (open circles) for the potential parameters used in Ref. 4 to mimic the multiphoton ionization of Xe.

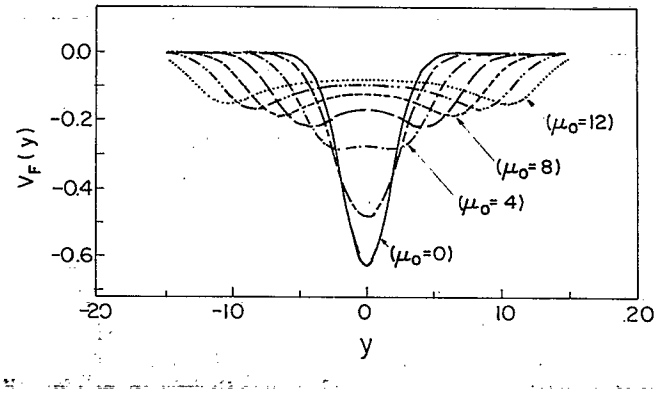

FIG. 3. The shape of the dressed-Gaussian potential as the field-strength parameter μ_0 (in atomic units) is varied.

plex eigenvalues of the evaluation operator for one optical cycle were obtained in five significant digits of accuracy.

III. THE PERIODICALLY DRIVEN GAUSSIAN POTENTIAL

The model Hamiltonian (an inverted-Gaussian potential) studied by Bardsley, Szöke, and Comella[4] for a Xe electron in radiation gause (for $\hbar=m=1$) is

$$\hat{H}(x,t)=\frac{1}{2}\left[\hat{p}-\frac{\epsilon_0}{\omega}\sin(\omega t)\right]^2-V_0\exp[-(x/x_0)^2]. \tag{14}$$

For $V_0=0.63$ a.u. and $x_0=2.65$ a.u., the zero-field potential (i.e., $\epsilon_0=0$) supports two bound states which mimic the lowest two electronic states of Xe $E_0=-0.4451$ a.u. and $E_1=-0.1400$ a.u. and a third weakly bound state with the energy of $E_2\simeq-0.000\,14$ a.u. These states are shown in Fig. 1 for $\omega=0.0925$ a.u. in the absence of an external field.

In the presence of an external field, these three bound states become metastable due to the interaction with the field. The eigenvalues corresponding to these resonance states are shown in Fig. 2 for maximum field intensity of $I_0=4.3\times10^{13}$ W/cm^2 $\{I_0=[c/(8\pi)]\epsilon_0^2\}$. These states were formerly reported by Bardlsey and co-workers.[5] The fourth "extra" resonance state reported by these authors has an extremely short lifetime and could only be observed by "zooming in" on the vicinity of the origin ($\lambda=0+i0$).

Let us try to analyze the appearance of extra resonance states in terms of the dressed potential. It has been shown for some model Hamiltonians that the dressed potential may support more bound states than the field-free potential. The existence and number of additional states depends on the intensity and frequency of the field.[4–8] By making use of the Kramers–Henneberger transformation,[14,16] the dressed inverted-Gaussian potential is given by

$$V_{\text{dressed}}(x)=-V_0\int_0^1\exp(-\{[x+\mu_0\cos(2\pi t')]/x_0\}^2)dt', \tag{15}$$

where $\mu_0=\epsilon_0/\omega^2$. The shape of the dressed potential depends on the value of μ_0 as shown in Fig. 3. The potential becomes shallower as μ_0 increases in a way which is similar to the model studied by Pont, Walet, and Gavrila,[15] by Bhatt, Piraux, and Burnett[4] and by Su and Eberly.[7] The typical double well potentials obtained in previous works[6,7,10,15] are seen as well. The bound states of the dressed inverted-Gaussian potential were calculated numerically and are presented in Fig. 4. As one can see from Fig. 4 a fourth bound state is "created" at $\mu_0\simeq6$ a.u. (maximum field intensity of 10^{14} W/cm^2).

Guided by the observation of an increase in the number of bound states in the dressed potential with the increase of field intensity, we now examine the behavior of resonance quasienergies as a function of field intensity. In Figs. 5(a)–5(f), we show the motion of the eigenvalues λ_α associated with the resonance states [obtained by Eq. (9)]

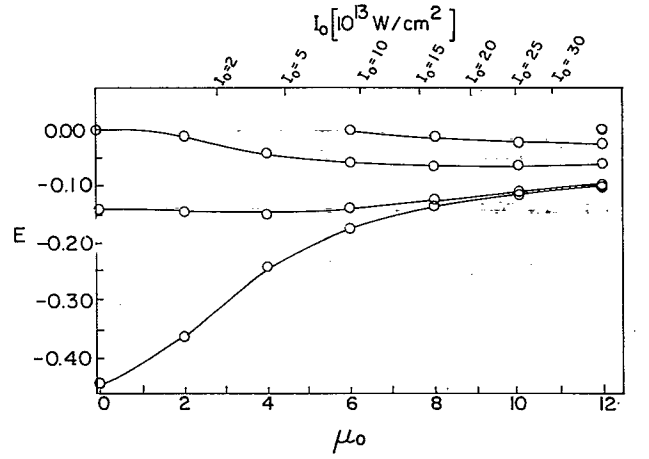

FIG. 4. The bound states of the dressed-Gaussian potential as a function of μ_0 (in atomic units) and of the field intensity I_0 (in units of 10^{13} W/cm^2).

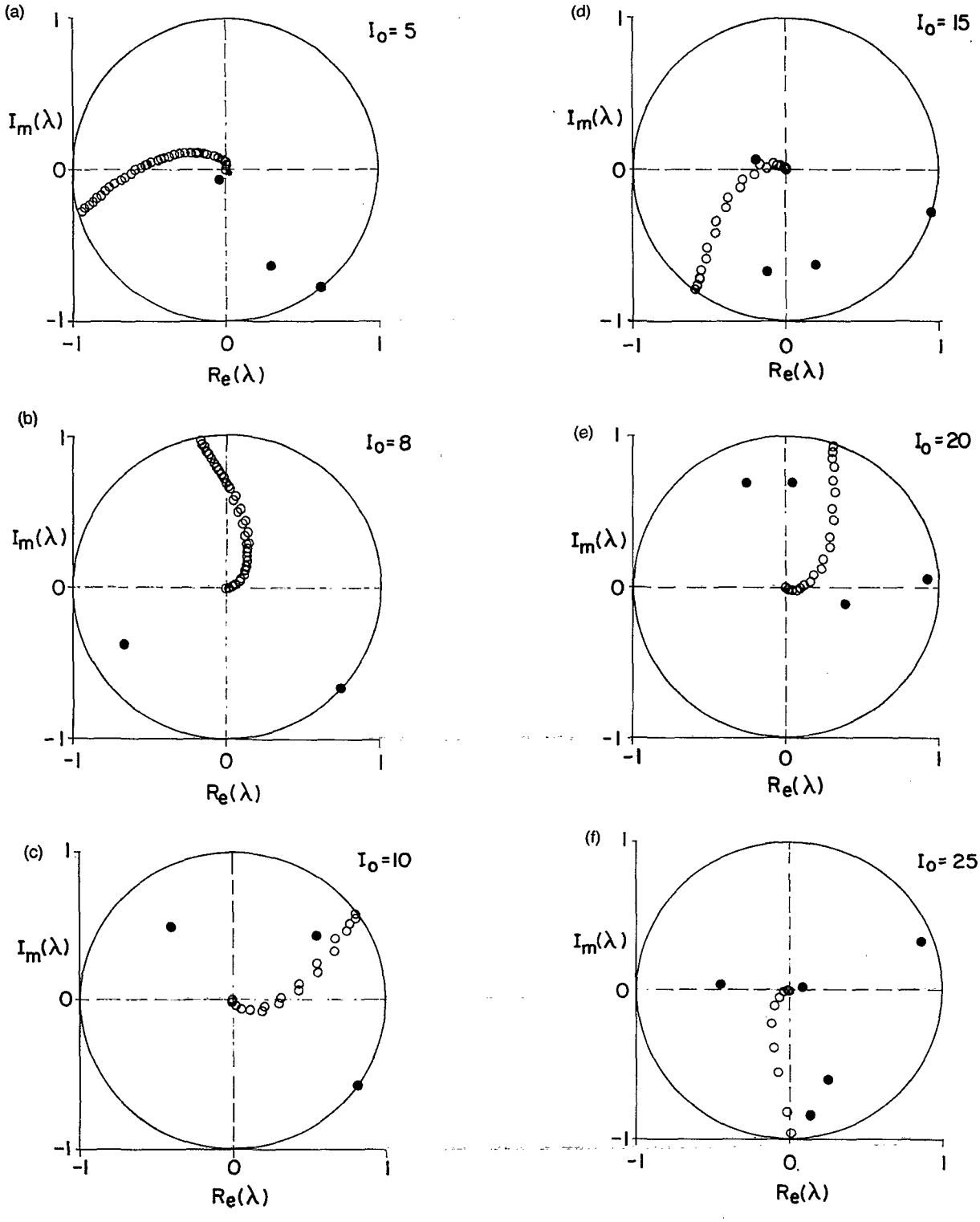

FIG. 5. The long-lived resonance quasienergies of the periodically driven Gaussian potential (full circles) obtained by the combination of the complex-coordinate method and Floquet theory (for $\theta=0.3$ and 0.5 rad), mapped onto the first optical cycle [Eq. (10)]. I_0 is the electromagnetic field intensity in units of 10^{13} W/cm^2.

in the complex λ plane as the field intensity is varied. One can see that the general trend (i.e., appearance of new resonance states) which is predicted by the dressed potential analysis (Fig. 4) is, indeed, manifested in Figs. 5(a)–5(f) also. In these figures, one can see that at $I_0=1.5\times10^{14}$ W/cm^2, four long-lived resonance states are obtained (the value predicted by the dressed potential analysis is $I_0=1\times10^{14}$ W/cm^2), and for $I_0\simeq2.5\times10^{14}$ W/cm^2, five long-lived resonances are obtained (the value predicted by the dressed potential analysis is 3.2×10^{14} W/cm^2).

A detailed comparison between Figs. 4 and 5(a)–5(f) shows, however, some discrepancies. There is no consis-

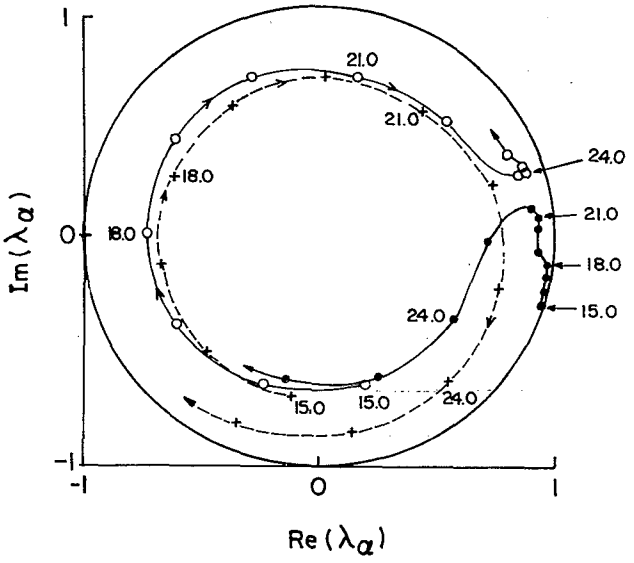

FIG. 6. Three resonance quasienergies $\lambda_\alpha=\exp(-i\epsilon_\alpha T/\hbar)$ of the periodically driven Gaussian potential as a function of the field intensity I_0 in units of 10^{13} W/cm^2. The three different most long-lived resonance quasienergy states are labeled, respectively, by ●, +, and ○.

tency in the gap between the value of the field intensity, I_0, at which a new long-lived discrete state appears and the value of I_0 predicted by the Kramers–Henneberger dressed states analysis. That is, the first new long-lived discrete state was obtained at I_0 which is larger than the predicted value of 10^{14} W/cm^2 and the second one was obtained at I_0 which is smaller than the predicted value. In addition, there is no way to predict, on the basis of the analysis of the energy spectrum of the dressed inverted-Gaussian potential, the drastic change in the lifetime of one out of the three discrete states as I_0 is varied. At $I_0=8\times10^{13}$ W/cm^2, one discrete state "disappears"—i.e., has a VERY short lifetime [see Fig. 5(b)]—but "suddenly" appears a long-lived state (not necessarily the same one) when $I_0=10^{14}$ W/cm^2 [see Fig. 5(c)].

Another discrepancy between the dressed potential predictions (Fig. 4) and the actual calculation (Figs. 5) is that except for the resonance states which have significantly long lifetimes such that they are visible in Fig. 5, there are also many short-lived resonances for which $\lambda_\alpha \simeq 0+i0$ such that they are not visible on the scale of Fig. 5. These short-lived resonance states do not appear in the dressed potential spectrum at all.

Figure 6 shows the dependence of three of the resonance quasienergies on the field intensity I_0 in the range of 15×10^{13}—26×10^{13} W/cm^2 for fixed frequency ($\omega=0.0925$ a.u.). Looking at this figure, one can see that up to the intensity of $I_0\simeq21\times10^{13}$ W/cm^2, the resonance quasienergy associated with the ground state of the field free Hamiltonian is rotated counterclockwise with the increase of field intensity. The quasienergies which correspond to the two other resonance states are rotated clockwise with the increase of field intensity. This phenomenon can be explained in terms of the bound states of the dressed potential. In Fig. 4, one can see that the energy of the ground state of the dressed potential is an increasing function of field intensity I_0, therefore, the zero-order resonance quasienergy $\lambda_0=\exp[-iE_0(I_0)T/\hbar]$ rotates counterclockwise with increasing I_0 (exactly as is the case in the full calculation shown in Fig. 6). The bound states of the dressed potential (Fig. 4) corresponding to these resonances are decreasing functions of I_0 such that their zero-order resonance quasienergies $\lambda_j=\exp[-iE_j(I_0)T/\hbar]$ ($j=1,2,3$) rotate clockwise with increasing I_0.

The agreement between the dressed potential prediction (Fig. 4) and the results of the full calculation (Fig. 6) breaks down for field intensities I_0 greater than 22×10^{13} W/cm^2, where a complicated avoided crossing event (for a detailed description, see Ref. 12) causes a drastic change in the behavior of resonance quasienergies as a function of the field intensity.

IV. THE PERIODICALLY DRIVEN MORSE POTENTIAL

The complex quasienergy of a driven Morse oscillator has been studied previously by Chu[11] for the study of two-photon dissociation of H_2^+ ions. Since we solve one electronic state problem, our formalism is simpler. The purpose of our study of the complex resonance quasienergy states of the periodically driven Morse oscillator is to investigate the sensitivity of creation and annihilation of discrete long-lived quasienergy states to the shape of the potential which describes a Xe electron in electromagnetic field.

The periodically driven Morse oscillator which describes a Xe electron in radiation gauge is given by

$$\hat{H}(y,t)=\frac{1}{2}\left[\hat{p}-\frac{\epsilon_0}{\omega}\sin(\omega t)^2\right]+D(e^{-2by}-2e^{-by}). \tag{16}$$

For $D=0.662\,033$ a.u. and $b=0.414\,354$ a.u., this field-free Hamiltonian has the same energy spectrum as does the field-free inverted-Gaussian Hamiltonian [Eq. (14)].

The quasienergy positions and widths of the periodically driven Morse potential were obtained by the combination of the complex coordinate method and Floquet theory as described in Sec. II (see also Ref. 13). The eigenvalues λ_α are presented in Figs. 7(a)–7(d) as a function of field intensity parameter $\mu_0=\epsilon_0/\omega^2\propto\sqrt{I_0}$. We will refer to the dependence of the resonance quasienergies on the field parameters (intensity and frequency) as resonance field dependence.

By comparing the resonance field dependence presented in Fig. 7 and the resonance field dependence obtained for the inverted Gaussian potential presented in Fig. 5, one can see that the three bound states of the two model Hamiltonians which have the same energies in the absence of an external field behave in an entirely different manner in the presence of strong laser fields. For the inverted-Gaussian model Hamiltonian (Fig. 5), one can see that the number of narrow quasienergy resonance states varies from 2 to 5 as I_0 is increased, whereas for the Morse po-

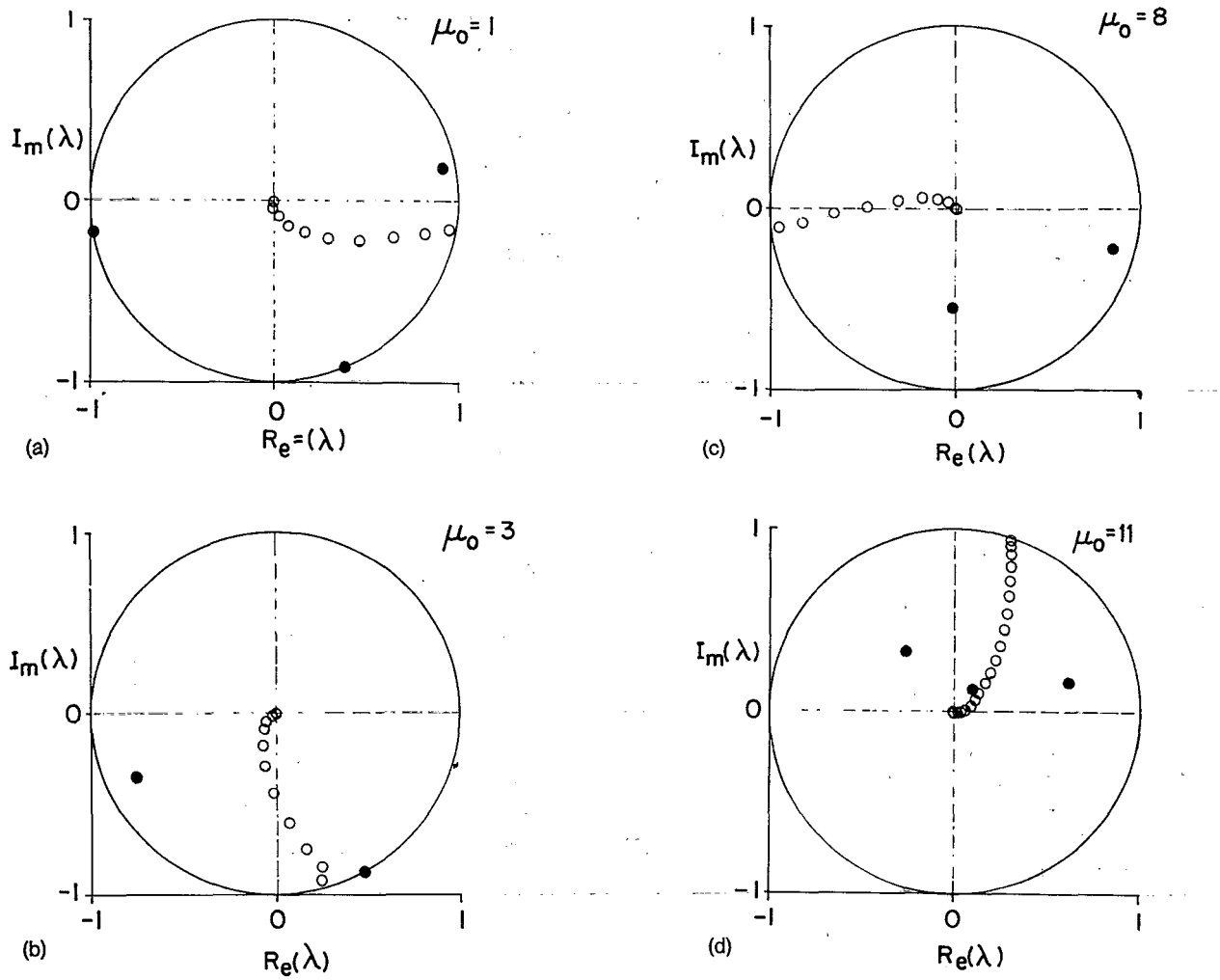

FIG. 7. The long-lived resonance quasienergies of the periodically driven Morse potential (full circles) obtained by the combination of the complex-coordinate method and Floquet theory, mapped to the unit circle [Eq. (10)]. μ_0 is the field-strength parameter (in atomic units).

tential (Fig. 7), one can see that the number of these states varies only from 2 to 3 as $I_0 \propto \mu_0^2$ is increased in the same range.

The observation described above suggests that tracing the behavior of resonance quasienergies as a function of field parameters such as frequency, intensity, etc. (i.e., obtaining the resonance field dependence) might help to determine which model potential is more suited for adequately describing the physical system under study.

The question arises whether the difference between the resonance field dependence of the two studied models can be explained on the basis of the Kramers–Henneberger dressed states analysis. We will show that the dependence of the bound energy levels of the dressed potential of the Morse and the inverted-Gaussian model Hamiltonians on the field strength amplitude is different for the two models. With increasing field strength, the number of bound states decreases for the Morse potential and increases for the inverted-Gaussian potential. Therefore, the general trends of the difference between the resonance field dependence of the two model systems can be explained on the basis of the dressed potentials analysis. Nevertheless, we will also show that the details of the resonance field dependence cannot be predicted in terms of the dressed potentials analysis alone.

Following the Kramers–Henneberger transformation[14,16] the dressed driven Morse potential is given by

$$V_{\text{dressed}}(y)=\int_0^1 V[y+\mu_0\cos(2\pi t')]dt', \tag{17}$$

where $V(x)=D\ (e^{-2bx}-2e^{-bx})$ and $\mu_0=\epsilon_0/\omega^2$. Therefore, the dressed potential is a Morse potential also

$$V_{\text{dressed}}(y)=\tilde{D}[(1+\delta)e^{-2by}-2e^{-by}], \tag{18}$$

where

$$\delta=A_0(2b\mu_0)/A_0(b\mu_0)-1,$$

$$\tilde{D}=DA_0(b\mu_0),$$

and A_0 is the modified Bessel function.

The shape of the dressed potential depends on the value of μ_0 as is shown in Fig. 8. However, unlike the models studied by Bhatt, Piraux, and Burnett,[4] by Bardsley and Commella,[8] and by Pont, Walet, and Gavrila,[15] the dressed Morse potential supports less bound states as μ_0 is increased. The eigenvalues of the dressed Morse potential

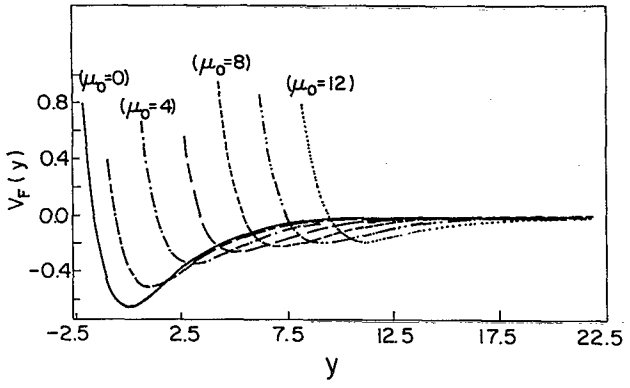

FIG. 8. The shape of the dressed-Morse potential as the field-strength parameter μ_0 (in atomic units) is varied.

are a special case of the generalized Morse potential which was solved analytically by Engdahl and Moiseyev.[18]

The eigenvalues of the dressed Morse potential are given by

$$E_n=-\frac{\tilde{D}}{1+\delta}\left[1-b\sqrt{\frac{1+\delta}{2\tilde{D}}}\left(n+\frac{1}{2}\right)\right]^2 \tag{19}$$

such that the number of bound states is given by

$$n\leqslant\frac{1}{b}\sqrt{\frac{2DA_0^2(b\mu_0)}{A_0(2b\mu_0)}}-\frac{1}{2} \tag{20}$$

since $A_0(b\mu)/A_0^{1/2}(2b\mu_0)<1$ for $\mu_0>0$, it is clear that the number of bound states is reduced as field intensity is increased. The analytical energy spectrum of the dressed Morse potential presented in Fig. 9 shows the annihilation of one bound state at $\mu_0>2$ a.u. and the annihilation of the second one at $\mu_0>11$ a.u. When carrying out actual calculations for this system, one finds out that indeed, as predicted by the analysis of the energy spectrum of the dressed Morse potential (Fig. 9) only two (out of three) long-lived quasienergy states [denoted by the full circles in Fig. 7(b)] were obtained for $\mu_0=3$ a.u. At $\mu_0=11$ a.u., however, contrary to the dressed potential analysis predicting the annihilation of an additional state, we find that an additional state is created such that three long-lived quasienergy states exist for this field amplitude. This phenomenon cannot be explained on the basis of Kramers–Henneberger analysis.

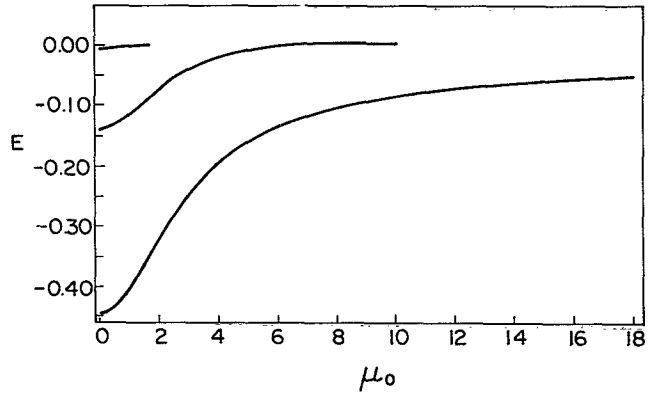

FIG. 9. The bound states of the dressed-Morse potential as a function of μ_0 (in atomic units).

We see that also for the Morse potential (as was the case for the inverted-Gaussian potential in Sec. III), the dressed potential spectrum can provide a basis for prediction of the resonance field dependence at relatively weak fields, but fails to do so for high fields.

V. CONCLUDING REMARKS

The sensitivity of creation and annihilation of discrete long-lived quasienergy states to the shape of the potential was studied. Two different model potentials—an inverted-Gaussian potential and a Morse potential—which mimic the lowest two electronic states of Xe were investigated. The dependence of the resonance positions and widths on the field strength in the two model systems was found to be dramatically different. Our conclusion is that resonance field dependence may provide unique information which can help determine the adequacy of a potential in representing a physical system under study. The physical interpretation of this result is clear. The knowledge of the energy spectrum, as is well known, does not provide sufficient information for the unique determination of the interaction potential. The dependence of the energy of the quasienergy spectrum on a potential parameter such as the field strength parameter provides sufficient information to solve the inverse problem and to determine the shape of the interaction potential.

It is shown that even for nonweak (but not too high) fields, the bound state energy levels of the dressed potential [Eq. (17)] are in agreement with the corresponding complex scaled quasienergies as noticed before by Bardsley and Comella.[8] Moreover, one can predict the resonance field dependence by the study of the effect of the field intensity on the bound state energy spectrum of the dressed potential.

The dressed potential analysis can also provide clues for the possible occurrence of avoided crossing events. We have demonstrated above that the direction of rounding about of a specific resonance quasienergy in the complex λ plane as a function of increasing field intensity is governed by the increase/decrease of the corresponding bound state of the dressed potential. An avoided crossing event is more likely to occur between resonances having quasi-energies which are moving towards each other as a function of changes in the field parameters (frequency, intensity, etc.). On this basis, information on the behavior of the bound states of the dressed potential as a function of field intensity might help in the prediction of avoided crossing events. As mentioned, the behavior of systems exposed on an external field can be well predicted, for not very high field intensities, on the basis of an analysis of the bound states of the dressed potential of the system. This representation, therefore, provides a much better zero-order picture for systems interacting with periodic fields than do the interactionless potentials. For extremely strong fields, however, the actual

behavior of systems studied deviated markedly from the predictions made on the basis of the dressed potential analysis.

Finally, the combination of the Floquet and complex scaling theories enable an isolation of the metastable quasienergy states (i.e., resonances) from the scattering states as first discussed by Chu and Reinhardt.[2(a)] Here we show (see Figs. 1, 2, and 5–7) that one can use this property as a visual tool to observe the dependence of resonance quasienergies on field intensity and frequency.

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